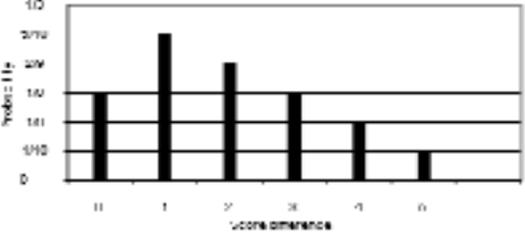
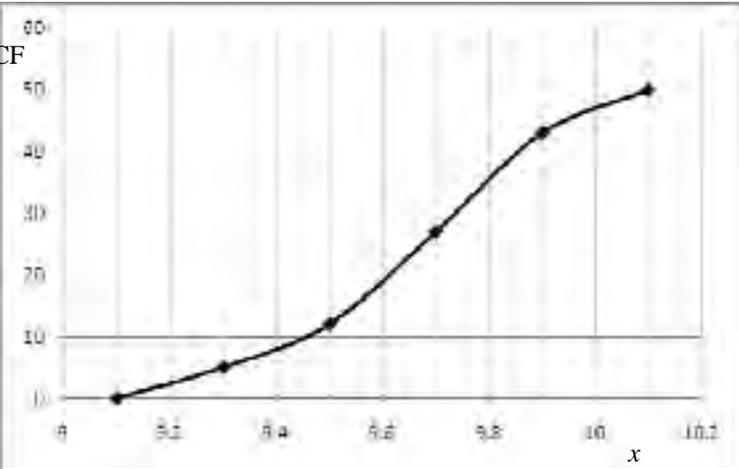


1 (i)	$1000 \times 0.013 = 13$ Or $0.2 \times 65 = 13$ Or $0.2 \times 5 \times 13 = 13$	M1 A1 M1 for 0.2×65	2	Allow with or without working For MR $1000 \times 0.13 = 130$ Allow M1A0 Allow M1A0 if extra terms added eg 1000×0.004 SC1 for $1000 \times 0.014 = 14$ For whole calculation
(ii)	Positive	B1	1	Allow +ve but NOT skewed to the right Do not allow 'positive correlation'
(iii)	Minimum value = 1500 Maximum value = 2500	B1 Without wrong working B1 Without wrong working	2	Exact answers only unless good explanation such as eg no road has length zero so min is eg 1501 SC1 for lower answer between 1499 and 1501 and upper between 2499 and 2501 Allow answer given as inequality
		TOTAL	5	

<p>2 (i)</p>			<p>G1 labelled linear scales on both axes G1 heights</p>	<p>2</p> <p>Accept r or x for horizontal label and p or better for vertical including probability distribution Visual check only Allow G1G0 for points rather than lines Bars must not be wider than gaps for second G1 Condone vertical scale 1, 2, 3, 4, 5 and Probability (\times) 1/18 as label BOD for height of $r = 0$ on vertical axis</p>
<p>(ii)</p>	<p>(A) If $X = 1$, possible scores are (1,2), (2,3), (3,4), (4,5), (5,6) and (2,1), (3,2), (4,3), (5,4), (6,5)</p> <p>(All are equally likely) so probability = $\frac{10}{36} = \frac{5}{18}$</p> <p>(B) If $X = 0$, possible scores are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) so probability = $\frac{6}{36} = \frac{1}{6}$</p>	<p>M1 A1 B1</p>	<p>2</p> <p>Also M1 for a clear correct sample space seen with the ten 1's identified by means of circles or ticks or soi. Must be convincing. No additional values such as 0,1 and 1,0 Do not allow 'just 10 ways you can have a difference of 1 so 10/36' or equivalent SC1 for possible scores are (1,2), (2,3), (3,4), (4,5), (5,6) so probability = $2 \times 5 \times 1/36$ with no explanation for $2 \times$</p> <p>1</p> <p>Also B1 for a clear correct sample space seen with the six 0's identified by means of circles or ticks or soi. Must be convincing. No additional values. Allow both dice must be the same so probability = $6/36 = 1/6$. Allow $1 \times 1/6 = 1/6$ BOD</p>	
<p>(iii)</p>	<p>Mean value of $X =$</p> $0 \times \frac{1}{6} + 1 \times \frac{5}{18} + 2 \times \frac{2}{9} + 3 \times \frac{1}{6} + 4 \times \frac{1}{9} + 5 \times \frac{1}{18} = 1 \frac{17}{18} = 1.94$	<p>M1 for $\sum rp$ (at least 3 terms correct) A1 CAO</p>	<p>2</p> <p>Or 35/18 Division by 6 or other spurious factor gets MAX M1A0</p>	
			<p>TOTAL 7</p>	

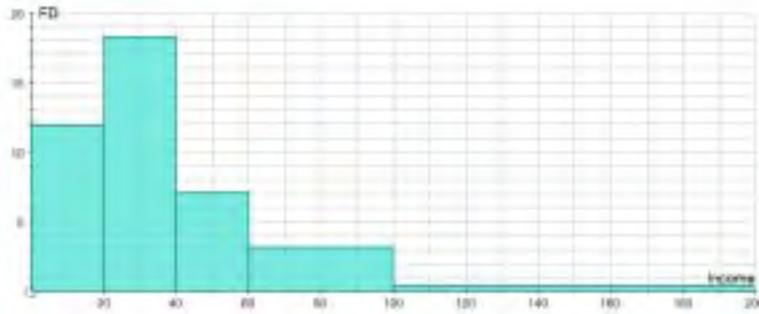
<p>3 (i)</p>	<table border="1" data-bbox="219 178 958 269"> <tr> <td>Upper Bound</td> <td>9.1</td> <td>9.3</td> <td>9.5</td> <td>9.7</td> <td>9.9</td> <td>10.1</td> </tr> <tr> <td>Cumulative frequency</td> <td>0</td> <td>5</td> <td>12</td> <td>27</td> <td>43</td> <td>50</td> </tr> </table> 	Upper Bound	9.1	9.3	9.5	9.7	9.9	10.1	Cumulative frequency	0	5	12	27	43	50	<p>B1 for cumulative frequencies</p> <p>G1 for scales</p> <p>G1 for labels</p> <p>G1 for points (Provided plotted at correct UCB positions)</p> <p>G1 for joining points</p> <p>All G's dep on attempt at cumulative frequency but not cumulative fx's or other spurious values.</p>	<p>May be implied from graph. Condone omission of 0 at this stage.</p> <p>Linear horizontal scale. Linear vertical scale: 0 to 50 (no inequality scales - Not even <9.1, <9.3, $<9.5 \dots$)</p> <p>Heating quality or x and Cumulative frequency or just CF or similar but not just frequency or fd nor cumulative fd</p> <p>5 Plotted as (UCB, their cf). Ignore (9.1,0) at this stage. No midpoint or LCB plots. Plotted within $\frac{1}{2}$ small square</p> <p>For joining all of 'their points' (line or smooth curve) AND now including (9.1,0) dep on previous G1</p> <p>Mid point or LCB plots may score first three marks</p> <p>Can get up to 3/5 for cum freq bars Allow full credit if axes reversed correctly</p> <p>Lines of best fit could attract max 4 out of 5.</p>
Upper Bound	9.1	9.3	9.5	9.7	9.9	10.1											
Cumulative frequency	0	5	12	27	43	50											
<p>(ii)</p>	<p>Median = 9.67</p>	<p>B1 FT Allow answers between 9.66 and 9.68 without checking curve. Otherwise check curve.</p>	<p>3 Based on 25th to 26th value on a cumulative frequency graph ft their mid-point plot (not LCB's) approx 9.57 for m.p. plot Allow 9.56 to 9.58 without checking B0 for interpolation</p>														

	$Q1 = 9.51$ $Q3 = 9.83$ Inter-quartile range = $9.83 - 9.51 = 0.32$	B1 FT for Q3 or Q1 B1 FT for IQR providing both Q1 and Q3 are correct Allow answers between 9.50 and 9.52 and between 9.82 and 9.84 without checking curve. Otherwise check curve.		Based on 12 th to 13 th and 37 th to 38 th values on a cumulative frequency graph fit their mid -point plot (not LCB's) approx $Q1 = 9.42$; $Q3 = 9.73$ Allow 9.41 to 9.43 and 9.72 to 9.74 without checking B0 for interpolation Allow correct IQR from graph if quartiles not stated Lines of best fit: B0 B0 B0 here.
(iii)	Lower limit $9.51 - 1.5 \times 0.32 = 9.03$ Upper limit $9.83 + 1.5 \times 0.32 = 10.31$ Thus there are no outliers in the sample.	B1 FT their Q_1 , IQR B1 FT their Q_3 , IQR E1 NB E mark dep on both B marks	3	Any use of <u>median</u> ± 1.5 IQR scores B0 B0 E0 If FT leads to limits above 9.1 or below 10.1 then E0 No marks for ± 2 or 3 IQR In this part FT their values from (ii) if sensibly obtained (eg from LCB plot) or lines of best fit, but not from location ie 12.5, 37.5 or cumulative fx's or similar. For use of mean $\pm 2s$, Mean = 9.652, s = 0.235, Limits 9.182, 10.122 gets M1 for correct lower limit, M1 for correct upper limit, zero otherwise, but E0 since there could be outliers using this definition
(iv)	(A) $P(\text{All 3 more than 9.5}) = \frac{38}{50} \times \frac{37}{49} \times \frac{36}{48} = 0.4304$ (=50616/117600 = 2109/4900)	M1 for $38/50 \times$ (triple product) M1 for product of remaining fractions A1 CAO	3	$(38/50)^3$ which gives answer 0.4389 scores M1M0A0 so watch for this. M0M0A0 for binomial probability including 0.76^{100} but ${}^3C_0 \times 0.24^0 \times 0.76^3$ still scores M1 $(k/50)^3$ for values of k other than 38 scores M0M0A0 $\frac{k}{50} \times \frac{(k-1)}{49} \times \frac{(k-2)}{48}$ for values of k other than 38 scores M1M0A0 Correct working but then multiplied or divided by some factor scores M1M0A0

	<p>(B) $P(\text{At least 2 more than 9.5}) = 3 \times \frac{38}{50} \times \frac{37}{49} \times \frac{12}{48} + 0.4304$ $= 3 \times 0.1435 + 0.4304$ $= 0.4304 + 0.4304$ $= 0.861$ $(=101232/117600 = 4218/4900 = 2109/2450)$</p> <p>OR</p> <p>$P(\text{At least 2 more than 9.5}) = 1 - (P(0) + P(1))$ $= 1 - \left[\left(\frac{12}{50} \times \frac{11}{49} \times \frac{10}{48} \right) + \left(3 \times \frac{12}{50} \times \frac{11}{49} \times \frac{38}{48} \right) \right]$ $= 1 - [0.01122 + 0.12796] = 1 - 0.13918 = 0.861$</p>	<p>M1 for product of 3 correct fractions seen M1 for 3 × a sensible triple or sum of 3 sensible triples M1 indep for + 0.4304 FT (providing it is a probability) A1 CAO</p> <p>M1 for $12/50 \times 11/49 \times 38/48$ M1 for 3 × a sensible triple or sum of 3 sensible triples M1 dep on both previous M1's for $1 - [0.01122 + 0.12796]$ A1 CAO</p>	<p>4</p>	<p>Accept 0.43 with working and 0.430 without working Or $\binom{38}{3} / \binom{50}{3} = 2109/4900 = 0.4304$</p> <p>Allow unsimplified fraction as final answer 50616/117600</p> <p>Or $\binom{38}{2} \binom{12}{1} / \binom{50}{3} = 0.4304$ gets first two M1M1's</p> <p>SC1 for $3 \times \frac{38}{50} \times \frac{38}{50} \times \frac{12}{50}$ or other sensible triple and SC2 if this + their 0.4304 (= 0.8549) Allow 0.86 or 2109/2450 or 4218/4900, but only M3A0 for other unsimplified fractions</p> <p>Use of 1 – method ‘with replacement’</p> <p>SC1 for $3 \times \frac{12}{50} \times \frac{12}{50} \times \frac{38}{50}$</p> <p>SC2 for whole of $1 - 3 \times \frac{12}{50} \times \frac{12}{50} \times \frac{38}{50} + \frac{12}{50} \times \frac{12}{50} \times \frac{12}{50}$ $(= 1 - (0.1313 + 0.0138) = 1 - 0.1451 = 0.8549)$</p>
		<p>TOTAL</p>	<p>18</p>	

4
(i)

Income	Frequency	Width	FD
$0 \leq x \leq 20$	238	20	11.9
$20 < x \leq 40$	365	20	18.25
$40 < x \leq 60$	142	20	7.1
$60 < x \leq 100$	128	40	3.2
$100 < x \leq 200$	45	100	0.



INCORRECT DIAGRAMS:

Frequency diagrams can get M0, A0, L0, W1, H0
MAXIMUM

Thus frequency density = frequency \times width,
frequency/midpoint etc gets MAX M0A0L0W1H0

M1 for fds
A1 CAO

Accept any suitable unit
for fd such as eg freq
per £1000.

L1 linear scale and
label on vertical axis

W1 linear scale on
horizontal axis and
correct width of bars

H1 height of bars

5

At least 4 fds correct for M1

M1 can be also be gained from freq per 10K - 119,
182.5, 71, 32, 4.5 (at least 4 correct) and A1 for all
correct

Accept any suitable unit for fd, eg freq per £10K, BUT
NOT FD per £1000

Allow fds correct to at least one dp

If fd not explicitly given, M1 A1 can be gained from
all heights correct (within one square) on histogram
(and M1A0 if at least 4 correct)

Allow restart although given fd wrong

For L1, label required on vert axis in relation to first
M1 mark ie fd or frequency density or if relevant
freq/£10K, freq/£k etc (NOT fd/£10K)

Accept f/w or f/cw (freq/width or freq/class width)

Ignore horizontal label

L1 can also be gained from an accurate key – may see
1 square = 36.5 or 23.8 or 14.2

For W1, must be drawn at 0, 20, 40 etc NOT 19.5 or
20.5 etc NO GAPS ALLOWED

Must have linear scale.

No inequality labels on their own such as $0 \leq I < 20$,
 $20 \leq I < 40$ etc but allow if a clear horizontal linear scale
is also given.

FT of heights *dep* on M1 all must agree with their fds

If fds not given and one height is wrong then max
M1A0L1W1H0

– visual check only (with one square) –no need to
measure precisely

	$\text{Mean} = \frac{10 \times 238 + 30 \times 365 + 50 \times 142 + 80 \times 128 + 150 \times 45}{918}$ $= \frac{37420}{918} = 40.8$	<p>M1 for midpoints M1 for midpoints ×frequencies with divisor 918 A1 CAO</p>	3	
(iii)	$\sum fx^2 = 238 \times 10^2 + 365 \times 30^2 + 142 \times 50^2 + 128 \times 80^2 + 45 \times 150^2$ $= 2539000$ <p>Or $238 \times 100 + 365 \times 900 + 142 \times 2500 + 128 \times 6400 + 45 \times 22500 = 2539000$</p> <p>Or $2380 \times 10 + 10950 \times 300 + 7100 \times 50 + 10240 \times 80 + 13500 \times 150 = 2539000$</p> $S_{xx} = 2539000 - \frac{37420^2}{918} = 1013666$ $s = \sqrt{\frac{1013666}{917}} = 33.2$	<p>M1 for at least 3 multiples fx^2 A1 for $\sum fx^2$</p> <p>M1 for attempt at S_{xx} Dep on first M1 BUT NOTE M1M0 if their $S_{xx} < 0$</p> <p>A1 CAO If using LCB or UCB</p>	4	<p>For A1, all midpoints and frequencies correct</p> <p>Or $S_{xx} = 2539000 - 918 \times 40.76^2 = 1013855$, $s = 33.25$. Using mean 40.8 leads to 1010861, $s = 33.20$, Using mean = 41 leads to $S_{xx} = 995844$ and $s = 32.95$</p> <p>M1M1 for $\sum f(x-\bar{x})^2$ M1 for first three terms, M1 for all 5 terms</p> <p>$238 \times (10-40.76)^2 + 365 \times (30-40.76)^2 + 142 \times (50-40.76)^2 + 128 \times (80-40.76)^2 + 45 \times (150-40.76)^2 (= 1013666)$ A1 for $S_{xx} = 1013666$ A1 for final answer</p>

		consistently then allow SC2 if working is fully correct but SC0 otherwise but no marks in part (ii)		For answer 33.25 or 33.3 or 33.2 (www) can just mark as B4 - these may be from calculator without working Allow 33 with correct working $\text{rmsd} = \sqrt{(1013666/918)} (=33.23)$ gets M1A1M1A0 (if seen) WATCH FOR DIVISOR OF 918 Allow max 4 sf in final answer Allow £33200 etc
(iv)	$(\bar{x} - 2s = 40.76 - 2 \times 33.25 = -25.74)$ $\bar{x} + 2s = 40.76 + 2 \times 33.25 = 107.26$ Comment that there are almost certainly some outliers. Appropriate comment such as ‘No, since there is nothing to indicate that these high earners represent a separate population.’	M1 for $\bar{x} + 2s$ or $\bar{x} - 2s$ A1 for 107.26 (FT) E1 E1 Dep on upper limit in range 106 - 108	4	FT any positive mean and positive sd for M1 Only follow through numerical values, not variables such as s , so if a candidate does not find s but then writes here ‘limit is $40.76 + 2 \times \text{standard deviation}$ ’, do NOT award M1 (This rule of not following through variables applies in all situations) Award E0E0 if their upper limit > 200 Allow ‘Must be some outliers’ Allow any comments that implies that there are outliers No marks in (iv) unless using $\bar{x} + 2s$ or $\bar{x} - 2s$
(v)	New mean = $1.15 \times 40.76 = 46.87$ New variance = $1.15^2 \times 33.25^2 = 1462$ For misread 1.5 in place of 1.15 For $1.5 \times 40.76 = 61.1$ and $1.5^2 \times 33.25^2 = 2490$ allow SC2 if all present but SC0 otherwise	B1 FT M1A1 FT	3	FT their mean (if given to ≥ 2 s.f.) FT their s (if given to ≥ 2 s.f.) provided their $s > 0$ If RMSD found in part (i) rather than s , then FT their RMSD For new SD = 38.24 found instead of variance give M1A0 even if called variance (and FT their s) M0A0 for $1.15 \times 33.25^2 = 1271$ Allow max 4 sf in final answers Min 2 sf If candidate ‘starts again’ only award marks for CAO
		TOTAL	19	